

8.7.8

$$\begin{aligned} 5. \int_1^{\infty} 2x^{-2} dx \\ &= \lim_{b \rightarrow \infty} \int_1^b 2x^{-2} dx \\ &= \lim_{b \rightarrow \infty} -x^{-1} \Big|_1^b \\ &= 0 - (-1) = 1 \end{aligned}$$

$$\begin{aligned} 7. \int_0^{\infty} e^{-x} dx \\ &= \lim_{b \rightarrow \infty} \int_0^b e^{-x} dx \\ &= \lim_{b \rightarrow \infty} -\frac{1}{x} e^{-x} \Big|_0^b \\ &= 0 - (-\frac{1}{x}) = \frac{1}{x} \end{aligned}$$

$$\begin{aligned} 9. \int_{-2}^{\infty} \frac{1}{x+4} dx \\ &\text{令 } u = x+4 \Rightarrow du = dx \\ &= \int_{-2}^{\infty} \frac{1}{u} du \\ &= \lim_{b \rightarrow \infty} \int_{-2}^b \frac{1}{u} du \\ &= \lim_{b \rightarrow \infty} \ln|u| \Big|_{-2}^b \\ &= \lim_{b \rightarrow \infty} \ln|b| - \ln| -2| \end{aligned}$$

$$\begin{aligned} 11. \int_2^{\infty} \frac{1}{(x-2)x} dx \\ &\text{令 } u = x-2 \Rightarrow du = dx \\ &= \int_2^{\infty} u^{-\frac{2}{2}} du \\ &= \lim_{b \rightarrow \infty} -2u^{-\frac{1}{2}} \Big|_2^b \\ &= 0 - (-2) = 2 \end{aligned}$$

$$\begin{aligned} 13. \int_{-\infty}^0 \frac{x}{(x^2+1)^2} dx \\ &\text{令 } u = x^2+1 \Rightarrow du = 2x dx \\ &= \frac{1}{2} \lim_{b \rightarrow \infty} \int_{b^2+1}^1 \frac{1}{u^2} du \\ &= \frac{1}{2} \lim_{b \rightarrow \infty} -\frac{1}{u} \Big|_{b^2+1}^1 \\ &= \frac{1}{2} (-\frac{1}{1}) = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 15. \int_1^{\infty} \frac{x^2+x+1}{x^4} dx \\ &= \int_1^{\infty} (x^{-2} + x^{-3} + x^{-4}) dx \\ &= \lim_{b \rightarrow \infty} -x^{-1} - \frac{1}{2}x^{-2} - \frac{1}{3}x^{-3} \Big|_1^b \\ &= -(-1-2-3) = 6 \end{aligned}$$

$$\begin{aligned} 17. \int_0^{\infty} \frac{e^x}{(1+e^x)^2} dx \\ &\text{令 } u = 1+e^x \Rightarrow du = e^x dx \\ &= \lim_{b \rightarrow \infty} \int_2^b \frac{1}{u^2} du \\ &= \lim_{b \rightarrow \infty} -\frac{1}{u} \Big|_2^b \\ &= 0 - (-\frac{1}{2}) = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 19. \int_{-\infty}^{\infty} x e^{-x^2} dx \\ &= \int_{-\infty}^1 x e^{-x^2} dx + \int_1^{\infty} x e^{-x^2} dx \\ &\text{令 } u = -x^2 \Rightarrow du = -2x dx \\ &= \int_{-\infty}^1 -\frac{1}{2} e^u du + \int_{-\infty}^1 -\frac{1}{2} e^u du \\ &= \lim_{b \rightarrow \infty} \int_b^1 -\frac{1}{2} e^u du + \lim_{b \rightarrow \infty} \int_b^1 -\frac{1}{2} e^u du \\ &= 0 \end{aligned}$$

$$\begin{aligned} 21. \int_{-\infty}^{\infty} \cos 2t dt \\ &= \int_{-\infty}^1 \cos 2t dt + \int_1^{\infty} \cos 2t dt \\ &= \lim_{b \rightarrow \infty} \int_b^1 \cos 2t dt + \lim_{c \rightarrow \infty} \int_1^c \cos 2t dt \\ &= \lim_{b \rightarrow \infty} \frac{1}{2} \sin 2t \Big|_b^1 + \lim_{c \rightarrow \infty} \frac{1}{2} \sin 2t \Big|_1^c \\ &= 0 \end{aligned}$$

$$\begin{aligned} 23. \int_0^{\infty} \sin^2 \alpha d\alpha \\ &= \frac{1}{2} \int_0^{\infty} (1 - \cos 2\alpha) d\alpha \\ &= \frac{1}{2} \alpha \Big|_0^{\infty} - \frac{1}{4} \sin 2\alpha \Big|_0^{\infty} \\ &= \frac{1}{2} \alpha \end{aligned}$$

$$\begin{aligned} 25. \int_1^{\infty} \frac{1}{x^2+x} dx \\ &= \int_1^{\infty} \frac{1}{(x+\frac{1}{2})^2 - \frac{1}{4}} dx \\ &\text{令 } x+\frac{1}{2} = \frac{1}{2} \sec \theta \Rightarrow dx = \frac{1}{2} \tan \theta \sec \theta d\theta \\ &= \int \alpha \frac{1}{\frac{1}{4} \sec^2 \theta} \cdot \frac{1}{2} \tan \theta \sec \theta d\theta \\ &= \lim_{b \rightarrow \infty} \ln|\sec \theta + \tan \theta| \Big|_{\alpha}^b \\ &= \end{aligned}$$

$$\begin{aligned} 27. \int_{-\infty}^0 x e^{x^2} dx \\ &\text{令 } u = x^2, du = 2x dx \\ &\Rightarrow du = 2x dx, v = \frac{1}{2} e^{x^2} \\ &= \lim_{b \rightarrow \infty} (\frac{1}{2} e^{x^2} \Big|_b^0 - \frac{1}{2} \int_b^0 e^{x^2} dx) \\ &= \lim_{b \rightarrow \infty} (\frac{1}{2} e^{x^2} \Big|_b^0 - \frac{1}{4} e^{x^2} \Big|_b^0) \\ &= 0 - \frac{1}{4} - 0 = -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} 29. \int_1^{\infty} \frac{\ln x}{x} dx \\ &\text{令 } u = \ln x, dv = x^{-1} dx \\ &\Rightarrow du = \frac{1}{x} dx, v = \ln|x| \\ &= (\ln x)^2 \Big|_1^{\infty} - \int_1^{\infty} \ln x \cdot \frac{1}{x} dx \\ &\Rightarrow \int_1^{\infty} \frac{\ln x}{x} dx = (\ln x)^2 \Big|_1^{\infty} - \int_1^{\infty} \frac{\ln x}{x} dx \\ &\Rightarrow 2 \int_1^{\infty} \frac{\ln x}{x} dx = (\ln x)^2 \Big|_1^{\infty} \\ &\Rightarrow \int_1^{\infty} \frac{\ln x}{x} dx = \lim_{b \rightarrow \infty} \frac{1}{2} (\ln x)^2 \Big|_1^b \\ &\Rightarrow \frac{1}{2} \ln^2 b \end{aligned}$$

$$\begin{aligned} 33. \int_0^{\infty} e^{-\sqrt{y}} dy \\ &\text{令 } u = -\sqrt{y} \Rightarrow du = -\frac{1}{2} y^{-\frac{1}{2}} dy \\ &\int_0^{\infty} -2u e^u du \\ &\text{令 } t = u, dv = e^u du \\ &\Rightarrow dt = du, v = e^u \\ &= \lim_{b \rightarrow \infty} (t e^t \Big|_0^b - \int_0^b e^t dt) \\ &= \lim_{b \rightarrow \infty} (t e^t \Big|_0^b - e^t \Big|_0^b) \\ &= -2(0 - 0 - \infty - 1) = 2 \end{aligned}$$

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{dx}{4x^2+4x+5} \\ &= \int_{-\infty}^1 \frac{1}{4x^2+4x+5} dx + \int_1^{\infty} \frac{1}{4x^2+4x+5} dx \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4 \sec^2 \theta} \sec^2 \theta d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4 \sec^2 \theta} \sec^2 \theta d\theta \\ &= \frac{1}{4} \theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{4} \theta \Big|_{\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{1}{4} \alpha + \frac{\pi}{8} + \frac{\pi}{8} - \frac{1}{4} \alpha = \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} 4(x^2+x)+5 \\ 4(x^2+x+\frac{1}{4}+\frac{1}{4})+5 \\ 4(x+\frac{1}{2})^2+4 \\ (2x+1)^2+4 \\ \text{令 } 2\cos \theta = 2x+1 \Rightarrow \tan \theta = \frac{3}{2} \end{aligned}$$

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8. |

11. ✓

$$y' = x^{\frac{1}{2}}$$

$$= \int_0^2 \sqrt{1 + (x^{\frac{1}{2}})^2} dx$$

$$= \int_0^2 \sqrt{1 + x} dx$$

$$= \frac{2}{3} (1+x)^{\frac{3}{2}} \Big|_0^2$$

$$= \frac{2}{3} \cdot 3^{\frac{3}{2}} - \frac{2}{3} \cdot 1^{\frac{3}{2}}$$

$$= 2\sqrt{3} - \frac{2}{3}$$

12. ✓

$$y' = 2x(1+x^2)^{\frac{1}{2}}$$

$$= \int_0^1 \sqrt{1+4x^2(1+x^2)} dx$$

$$= \int_0^1 \sqrt{1+4x^2+4x^4} dx$$

$$= \int_0^1 2x^2+1 dx$$

$$= \frac{2}{3} x^3 + x \Big|_0^1$$

$$= \frac{2}{3} + 1 = \frac{5}{3}$$

13. ✓

$$y' = x^2 - \frac{1}{4} x^{-2}$$

$$= \int_1^2 \sqrt{1 + (x^2 - \frac{1}{4} x^{-2})^2} dx$$

$$= \int_1^2 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16} x^{-4}} dx$$

$$= \int_1^2 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16} x^{-4}} dx$$

$$= \int_1^2 (x^2 + \frac{1}{4} x^{-2}) dx$$

$$= \frac{1}{3} x^3 - \frac{1}{4} x^{-1} \Big|_1^2$$

$$= \frac{8}{3} - \frac{1}{8} - \frac{1}{3} + \frac{1}{4}$$

$$= \frac{7}{3} + \frac{1}{8} = \frac{59}{24}$$

15.

$$y' = \frac{1}{\sin 2x} \cos 2x \tan x = \frac{\cos 2x}{\sin 2x} = \cot 2x$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cot 2x dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1}{\tan 2x} dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1}{2 \tan x \cos x} dx$$

17. ✓

$$y' = \frac{1}{\sec x} \sec x \tan x = \tan x$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x} dx$$

$$= \int_0^{\frac{\pi}{4}} \sec x dx$$

$$= \ln |\sec x + \tan x| \Big|_0^{\frac{\pi}{4}}$$

$$= \ln(\sqrt{2} + 1) - \ln 1$$

$$= \ln(\sqrt{2} + 1)$$

18. ✓

$$x = \frac{1}{5} y^{\frac{2}{3}} - y^{\frac{1}{3}}$$

$$\Rightarrow x' = \frac{2}{5} y^{-\frac{1}{3}} - \frac{1}{3} y^{-\frac{2}{3}}$$

$$= \int_1^9 \sqrt{\left(\frac{2}{5} y^{-\frac{1}{3}} - \frac{1}{3} y^{-\frac{2}{3}}\right)^2 + 1} dy$$

$$= \int_1^9 \sqrt{\frac{4}{25} y^{-\frac{2}{3}} + \frac{1}{9} y^{-\frac{4}{3}} + 1} dy$$

$$= \int_1^9 \left(\frac{2}{5} y^{-\frac{1}{3}} + \frac{1}{3} y^{-\frac{1}{3}} \right) dy$$

$$= \frac{2}{5} \cdot \frac{3}{2} y^{\frac{2}{3}} + \frac{1}{3} \cdot \frac{3}{2} y^{\frac{2}{3}} \Big|_1^9$$

$$= \frac{1}{5} \cdot 3^2 + 3 - \frac{1}{5} - 1$$

$$= \frac{22}{5}$$